## Midterm Review

- 1. These slides and review points found at http://math.utoledo.edu/~dgajews/1180
- 2. Bring a photo ID card: Rocket Card, Driver's License

## Covers:

4.1 Graphs + Euler Paths
4.2 Traveling Salesman + Hamiltonian Paths
2.1 Sets
2.2 Set Theory
2.3 Set Operations
6.1 Number Theory
11.1 Voting Methods
11.2 Defects of Voting Methods
11.3 Weighted Voting Systems

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Know the basic vocabulary of the sections.

The test will be multiple choice.

The test will be like the online HW rather than the lab assignments.

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Graphs are made up of 2 parts:

v and e

A graph is **connected** if













A complete graph has		
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A Hamiltonian path goes through each vertex once.

So a Hamiltonian path might miss edges.

An Euler path goes through each edge once.

It might use the same vertex multiple times.

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## Pop quiz!!! **Representing Sets** Hamiltonian Paths visit each \_\_\_\_ exactly once. • Set-builder notation: 1) Circuit 2) Edge $C = \{x : x \text{ is a carnivorous animal}\}$ 3) Complete 4) Vertex 5) None of the above 'such that" 'the set "equals" "x is a carnivorous animal" "of all x" or "is" 10/07/13 19 20

 $\{1, 2, 3, 4, \dots 10\}$ A set with no entries is known as the<br/>empty set. It can also be written as<br/> $\varnothing$  $\{x \mid x \text{ is positive and even }\}$ The empty set is a subset of every set.<br/>It is not an element of every set, but here<br/>is an example:<br/> $A = \{\text{ Bob, 12345, }\emptyset, \text{ pumpkins }\}$ 

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Section 2.1, Slide 20

n(A) = the number of elements in set A A = { Bob, 12345,  $\emptyset$ , pumpkins } n(A) = B = { a, b, c, ... x, y, z } n(B) = C = { x | x is a day of the week } n(C) =

∉ means "is *not* an element of"
Pop Quiz!!!! A = { 1, 2, 3, 4, 5} B = { {1}, {2}, {3}, {4}, {5} }
1) 5 is an element of
2) {5} is an element of
3) {5} is a subset of

 $\in$  means "is an element of"















 $U = \{ 1, 2, 3, 4, 5, 6 \}$ A =  $\{ 1, 2, 3, 4 \}$ B =  $\{ 2, 4, 6 \}$ C =  $\{ 3, 4, 5 \}$ 

Pop Quiz!!! How many elements in  $A \cap B$  ?

Find (A U B) - C'

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a | b means "a divides b" 5 | 30 7 | 7 If a | b then b = a c where c is some other number. This is a factor of b. 20 | 1000 and 1000 / 20 = 50, so  ${}_{10}bQQ0 = 20 \times 50$ 34

A number who's factors are only 1 and itself is a **prime** number. 2, 3, 5, 7, 11, 13, 23, 29, 31, 37, etc

You can use a Sieve of Eratosthenes to find them.

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A **factor tree** splits a number into 2 factors at each step.

Example: 420

The prime factorization is the collection of  $_{10}$  the primes.  $_{36}$ 



Example 3 candidates, 5 voters. Voter 1: A B C Voter 2: B A C Voter 3: C A B Voter 4: B C A Voter 5: C B A	Pairwise Comparison Compare every pair of candidates. If one wins, they get 1 point, the other 0. If they tie, they both get ½ points. The candidate with the most points wins.
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Example 3 candidates, 5 voters. Voter 1: A B C Voter 2: B A C Voter 3: C A B Voter 4: B C A Voter 5: C B A	Fairness Conditions and Criteria for analyzing defects.         DEFINITION The Majority Criterion         If a majority of the voters rank a candidate as their first choice, then that candidate should win the election.         DEFINITION Condorcet's Criterion         If candidate X can defeat each of the other candidates in a head-to-head vote, then X is the winner of the election.
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DEFINITION Independence-of-Irrelevant-Alternatives Criterion         If candidate X wins an election, some nonwinners are removed from the ballot, and a recount is done, then X still wins the election.         DEFINITION The Monotonicity Criterion         If X wins an election and in a reelection all voters who change their votes only change their votes to favor X, then X also wins the reelection.         No current voting method satisfies all of these well-meaning conditions and criteria         10/07/13       47	<ul> <li>Pop quiz!!!!</li> <li>Which of these is not a voting method?</li> <li>1) Borda Count</li> <li>2) Plurality with Elmination</li> <li>3) Plurality</li> <li>4) Majority Criterion</li> <li>5) Pairwise Comparison</li> </ul>



Compute the Banzhaf Power Index for A, B, C.					
{A} 5 {B} 3 {C} 4 {A,B} 8 {A,C} 9 {B,C} 7 {A,B,C} 12	winning winning winning	critical A, B A, C A			
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Compute the Banzhaf Power Index for A, B, C.						
{A} {B}	5 3		critica	I		
{C} {A,B} {A,C} {B,C} {A,B,C}	4 8 9 7 12	winning winning winning	A, B A, C A	(Note: total critical voters is 5 = 3+1+1)		
A critical 3 times, B critical 1 time, C critical 1 time						
Banzhaf Power Indéx A : 3/(3+1+1) B : 1/(3+1+1) C : 1/(3+1+1)						